

# **Manipulatives in Secondary School Mathematics**

**Greisy Winicki Landman**

**Dept of Mathematics and Statistics**

**California State Polytechnic University,  
Pomona**

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**I hear and I forget.  
I see and I remember.  
I do and I understand.**

**Confucius**



Students of all ages need to do math to understand math.

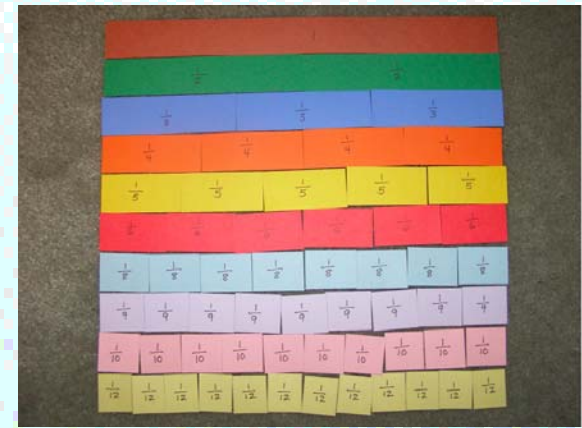
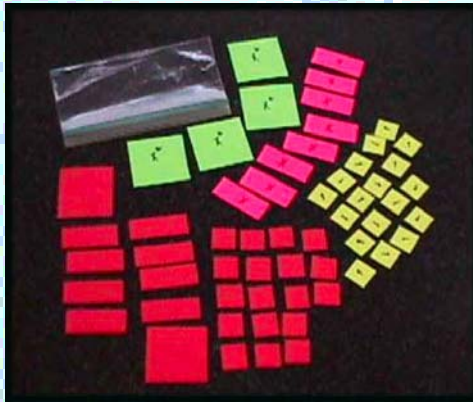
Manipulatives provide a way for students to do mathematics in a concrete manner and they learn some mathematics concepts better when explored with manipulatives.

Manipulatives are concrete objects that are commonly used in teaching mathematics.

They include attribute blocks, geometric shapes of different colors and sizes that may be used in classification or patterning tasks; colored counters for modeling addition and subtraction of integers; base ten blocks for representing and performing operations on decimal numbers; fraction pieces; algebra tiles, etc



Many different kinds of manipulatives are commercially available, and it is also possible to make them using common objects, such as craft sticks, beans, or buttons



Learning is enhanced when a concept is exposed to the students in a variety of contexts.

Active involvement is emphasized when students construct their own knowledge using a variety of tools and manipulatives.

Middle-school students are in transition between Piaget's **CONCRETE OPERATIONAL** and **FORMAL OPERATIONAL** stages of cognitive development. Many middle school students cannot completely understand concepts when presented to them in an abstract way only.

School teachers sometimes fail to see the purpose of manipulatives, citing reasons as time constrains and management problems, and generally feel that they are not important.

There can be some pitfalls to manipulatives, especially for struggling students.

Manipulatives are potentially confusing if their presentation is haphazard, disorganized, or lacking appropriate guidance and instruction from the teacher.

They can result in considerable time spent off-task or on activities that are not directly relevant to the needs of certain children.

Some children find concrete manipulatives a source of distraction and may do much better with visual or pictorial representations.

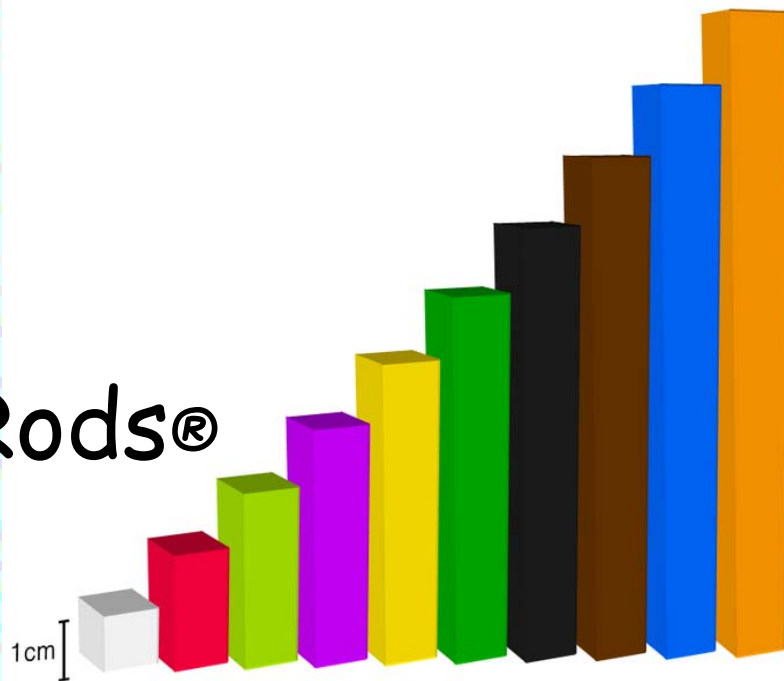
Monitoring the performance of groups of children may also be easier when visual or pictorial representations are used.



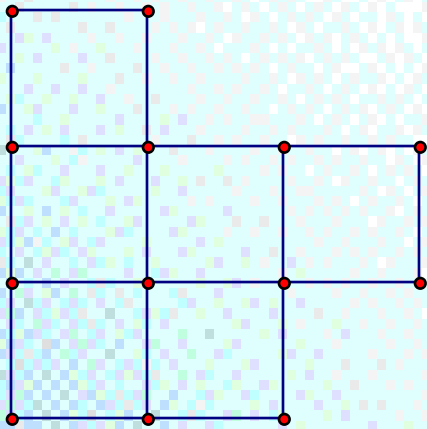


Polydron ®

Cuisenaire Rods®



DEF: A hexomino is a polyomino of order 6, that is, a planar polygon made of 6 congruent squares connected by their sides.

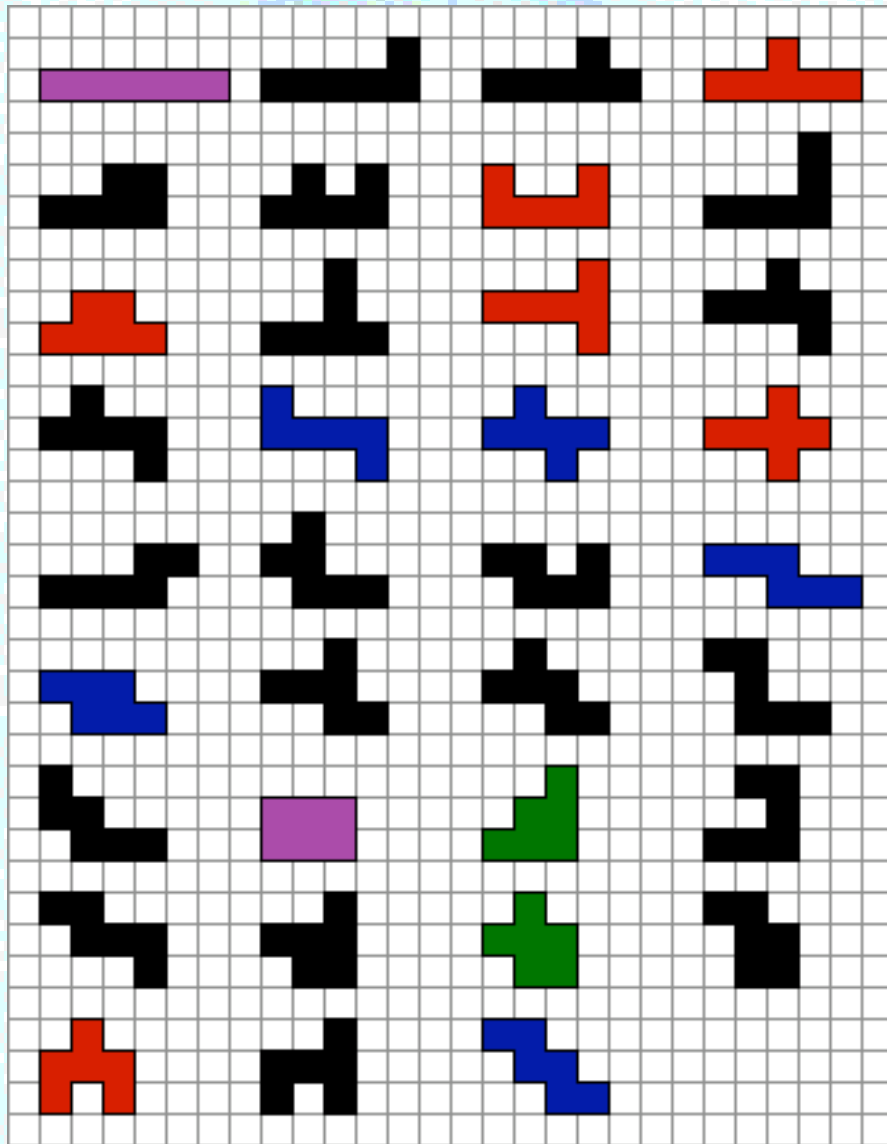


Example of a hexomino

Q1: How many hexominoes do exist?

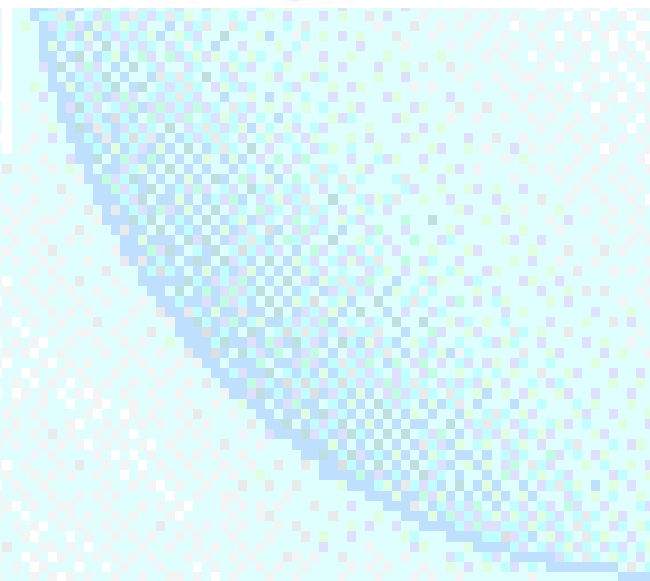
Q2: Which hexominoes fold into a cube?

Total: **35** hexominoes. Only **11** of them fold into a cube.



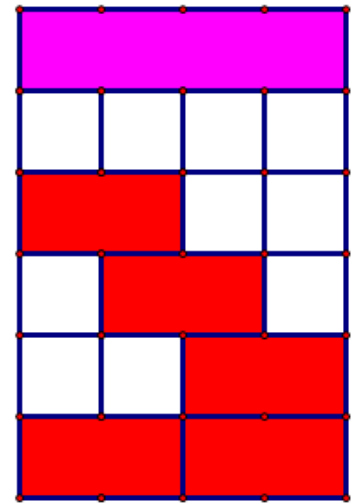
- 20 hexominoes (coloured black) have no symmetry.
- 6 hexominoes (coloured red) have exactly one symmetry line, parallel to the gridlines.
- 2 hexominoes (coloured green) have a symmetry line at  $45^\circ$  to the gridlines.
- 5 hexominoes (coloured blue) have only rotational symmetry.
- 2 hexominoes (coloured purple) have two symmetry lines - both parallel to the gridlines - and rotational symmetry.





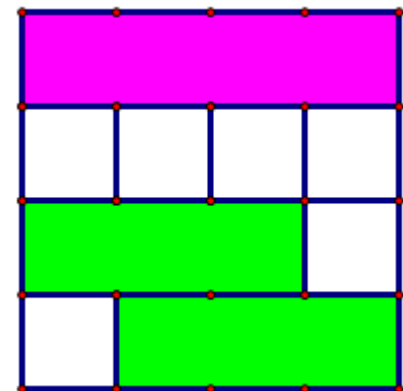
## Making trains:

Using only the red (2) and white (1) rods, how many different ways are there to make up a train of length 1? 2? 3?.... 10?.... n?



All the trains of length 4

Using only the light green (3) and white (1) rods, how many different ways are there to make up a train of length 1? 2? 3?.... 10?.... n?



All the trains of length 4